

UNIFORM SEDIMENT FLUX OR SEDIMENT BALANCE

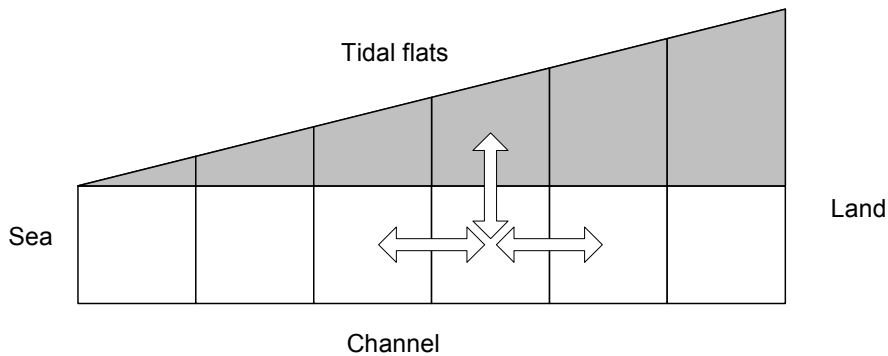
Method Indicator		
Bottom-Up	Hybrid	Top-Down
	YES	

Summary of key issues

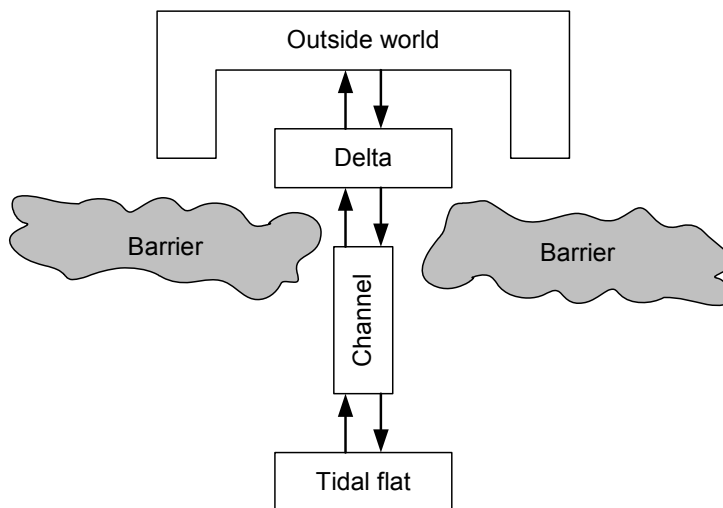
Issue	Summary
Description	In this type of model, sediment is moved within the estuary until a steady state is achieved when equal amounts on the flood and ebb tide.
Temporal Applicability	Typically applied over the long-term (10-100 years).
Spatial Applicability	Most often applied over the whole estuary as discrete areas of particular interest (e.g. tidal lagoon, channel, saltmarsh etc).
Links with Other Tools	Most recently applied to the ASMITA (Aggregated Scale Morphological Interaction between Tidal inlet and Adjacent coast) developed under ERP Phase 2.
Data Sources	An understanding of the sediment dynamics including: <ul style="list-style-type: none"> • Density; • Size; • Diffusion coefficients; • Fall velocity; • Hydrodynamic information (Flow speeds, water levels).
Necessary Software Tools / Skills	A top-down sediment flux/balance model and access to hydrodynamic information. The modelling requires a good understanding of the sediment transport regime within the estuary, long term evolution based on the equilibrium morphological state.
Typical Analyses	Changes in estuary hypsometry (area and shape).
Limitations	An understanding of equilibrium regime theory based on the balance of sediment flux for a particular estuary or environment is extremely complicated and requires specialist knowledge. Calibration and interpretation of model results is limited to historic evidence based on previous bathymetry.
Example Applications	Humber Estuary

Introduction

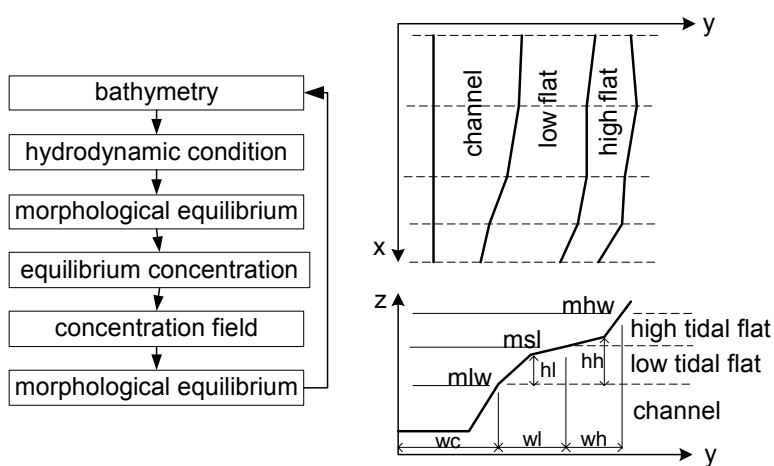
One of the earliest models to use the balance of net transport was a zero-dimensional model of a tidal lagoon by di Silvio (1989). The lagoon was schematised into just three components, tidal flat, littoral and tidal channel and the model was based on the idea that although locally and at any instant sediment transport is not in equilibrium, averaged over the system as a whole and a long enough period a relationship exists between the hydrodynamic quantities and the sediment concentrations. A slightly more detailed representation (see schematisation in Figure 1a) is used by van Dongeren and de Vriend (1994) for a 1-D model of an estuary, where the balance of sediment supply and demand is based on regime definitions for the channel and tidal flats.



a) Tidal basin schematisation used by van Dongeren and de Vriend (1994)



b) Schematisation of elements and exchanges within the ASMITA model



c) Procedure and schematisation used in ESTMORF (Wang et al, 1998)

Figure 1. Types of schematisation used in aggregated models

Using a similar concept but with a more complete representation of the estuary cross-sectional form Wang *et al.* (1998; 1999) proposed the ESTMORF model. This follows a similar approach to di Silvio in adopting a characteristic sediment concentration in each element of the model. At equilibrium it is assumed that there is no deposition or erosion anywhere in the model domain. If the sediment is mainly transported in suspension, the sediment flux is likely to be proportional to the flow field, so that the ratio between sediment flux and flow rate can be treated as a local equilibrium sediment concentration. Changes occur when the local sediment concentration is not equal to its local equilibrium value (see computational procedure in Figure 1c). The equilibrium concentration is determined from the equilibrium morphological state, rather than the hydrodynamic parameters, where the equilibrium state is defined using regime relationships for the channel and height of the tidal flats (see schematisation in Figure 1c). This model has now been widely applied to the study of long-term change in estuaries, as illustrated in Wang *et al.* (2000), [EMPHASYS Report](#) (see also Jeuken *et al.*, 2003).

Aggregation of the component parts of the tidal basin and adjacent coast, to a simple box model, (Figure 1b) has been developed as the ASMITA model by Stive *et al.* (1998) and Kragtwijk *et al.*, 2004). As with di Silvio's approach the model assumes that for each component there is a morphological equilibrium that depends on the hydrodynamic conditions. This allows the model to describe the evolution of the tidal inlet system towards equilibrium as a function of:

- The surplus or deficit of sediment,
- The capacity for exchange between the component parts, and
- The potential for sediment to be imported or exported across the model boundary.

This model has been applied to study the study of sea level rise on tidal basins (van Goor *et al.*, 2001) and the development of shorelines and tidal inlets over millennial time scales (Niedoroda *et al.*, 2003). The behaviour of systems has been outlined in greater detail in the Behaviour Systems supporting document.

Area and volume changes in an estuary

ASMITA (Aggregated Scale Morphological Interaction between a Tidal Basin and the Adjacent coast; Stive *et al.*, 1998) is an approach developed for the study of estuary response to sea level rise. The model is based on an aggregated model that considers the estuary as a box (or set of boxes) that fills and empties under tidal action. The methodology was developed to look at the combined response of tidal delta, channel and tidal flats and has been applied to the impact of sea level rise on tidal inlets (van Goor *et al.*, 2003) and the response of an estuary to the nodal tidal cycle (Jeuken *et al.*, 2003).

This section summarises the basis of the ASMITA model, combining multiple elements with marine and fluvial sources. In addition the existing ASMITA, which only considers variations in estuary volume, is extended to represent time varying surface area. This allows perturbations to be introduced in the form of changes to the volumes and areas due to interventions such as dredging and reclamation, as well as sea level rise and cyclic variations in the tidal range. The basis of the empirical relationship used to define the equilibrium relationships is also given some further consideration.

Single element volume model

For a single element model, comprising just an estuary channel, the equilibrium state is derived from the equilibrium relationship assumed between the channel volume and the tidal prism:

$$V_{ce} = f(P) \quad (1)$$

Where V_{ce} is the equilibrium volume of the channel and P is the tidal prism (Eysink, 1990).

The other assumption made is that the ratio of the actual flow velocity to the equilibrium condition is proportional to the ratio of the equilibrium volume and actual volume. The local equilibrium concentration can therefore be written in terms of the actual volume, V , and the equilibrium volume, V_{ce} :

$$c_{ce} = c_E \left(\frac{V_{ce}}{V} \right)^n \quad (2)$$

Here c_{ce} is the local equilibrium concentration, n is the concentration transport exponent and c_E is the equilibrium concentration for the system as a whole (usually taken as the open boundary value). The difference between the actual concentration and the local equilibrium value induces morphological change governed by the exchange between the water column and the bed:

$$\frac{dV}{dt} = wS(c_{ce} - c) \quad (3)$$

Where w is the vertical exchange coefficient, S is the surface or plan area of the channel and c is the actual concentration. If we now consider a sediment balance where the (long-term residual) sediment transport between elements is assumed to be simply diffusive (in this case the channel and the external environment):

$$wS(c_{ce} - c) = \delta_{co}(c - c_E) \quad (4)$$

The parameter δ_{co} is the horizontal exchange coefficient and the r.h.s. of equation 4 represents the exchange with the external environment.

Combining equations 2 to 4 and adopting a simple linear relationship for equation 1 of the form $V_e = \alpha P$, where α is an empirical coefficient, the moving surface volume, V_m , is given by:

$$\frac{dV_m}{dt} = \frac{w\delta c_E S}{\delta + wS} \left[\left(\frac{V_e(t)}{V_m(t)} \right)^n - 1 \right] + \frac{d\zeta}{dt} \cdot S + i\omega \frac{\eta}{2} e^{i\omega t} \cdot S \quad (5)$$

which includes the variation in volume due to sea level rise and the some cyclic variation in tidal range, such as the nodal tidal cycle.

In this form, the equilibrium volume, V_e , and the fixed surface volume, V_f can be written:

$$\begin{aligned} V_e(t) &= \alpha \cdot P = V_o + \alpha \cdot \Delta P \\ \Delta P &= \eta \cdot e^{i\omega t} \cdot S \\ V_f(t) &= V_m(t) - \frac{d\zeta}{dt} \cdot S \cdot t - \frac{\eta}{2} e^{i\omega t} \cdot S \end{aligned} \quad (6)$$

where the variables are defined as follows:

V_e	equilibrium volume;
V_o	initial volume at time $t=0$;
V_m	moving surface volume;
V_f	fixed surface volume;
P	tidal prism;
ζ	vertical sea level movement;
ω	angular frequency of nodal tide;
η	amplitude of nodal tide;
S	surface area of basin;
n	concentration transport exponent, usually taken as ~ 2 ;
w	vertical exchange rate;
δ	horizontal exchange rate;
C_E	global equilibrium concentration.

In this simple model the plan area is treated as constant. On the basis that $V_e \sim fn(P)$ any hydraulic changes that result in a change in tidal prism are represented in V_e but those that change the volume of the system (such as sea level rise, dredging or reclamation) are represented in V_m .

The variation in equilibrium, moving and fixed estuary volumes, due to a linear rise in sea level and the nodal tidal cycle, are illustrated in Figure 2. The lag and damping of the response, relative to the equilibrium volume, is due to the dynamics associated with the morphological response time of the estuary (Jeuken *et al.*, 2003).

The reduction of the volume relative to a fixed surface, shown in Figure 1, reflects the infilling of the basin that takes place, in order for the morphology to warp up vertically to keep pace with sea level rise (and the superimposed nodal cycle). Thus, vertical translation of the system is incorporated in this model. If, however, we wish to include the possibility of horizontal translation, as well as the vertical response, it is necessary to adjust the plan area rather than treat it as constant.

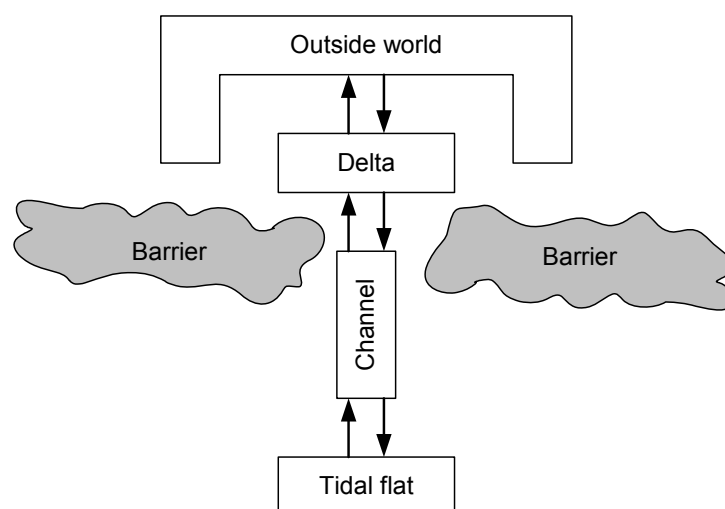


Figure 2. Variation in estuary volume for rising sea level and nodal tidal cycle as predicted by the single element model

Multi-element volume model

The basic concept of the multi-element model is to subdivide the estuary into a number of elements and define the exchanges between elements and the equilibrium conditions for each element. As already noted, this has been presented in the literature as the ASMITA model (Aggregated Scale Morphological Interaction between a Tidal basin the Adjacent coast) and was introduced by Stive *et al.* (1998; see also van Goor *et al.*, 2003; and Kragtwijk *et al.*, 2004). The system can be schematised into any number of discrete elements, which might be sections along the channel as used in ESTMORF (Wang *et al.*, 1998), or geomorphological components, such as the channel and tidal flats as typically used in ASMITA. This is illustrated in Figure 3, which shows the linkage between tidal flat, channel and tidal delta through to the open sea, referred to as the outside world. For each component the volume can be defined in terms of the sediment or water volume. In the derivation presented here, only water volumes are used and the equations presented reflect this (for the more general case see the papers noted above). So, for example, the scheme shown in Figure 3 would be represented by:

- Tidal delta - total water volume over delta (where the delta has a volume relative to undisturbed coastal bed);
- Channel - total water volume below MLW;
- Tidal flats - total water volume between MLW and MHW over flats (i.e. not including the prism over the channel).

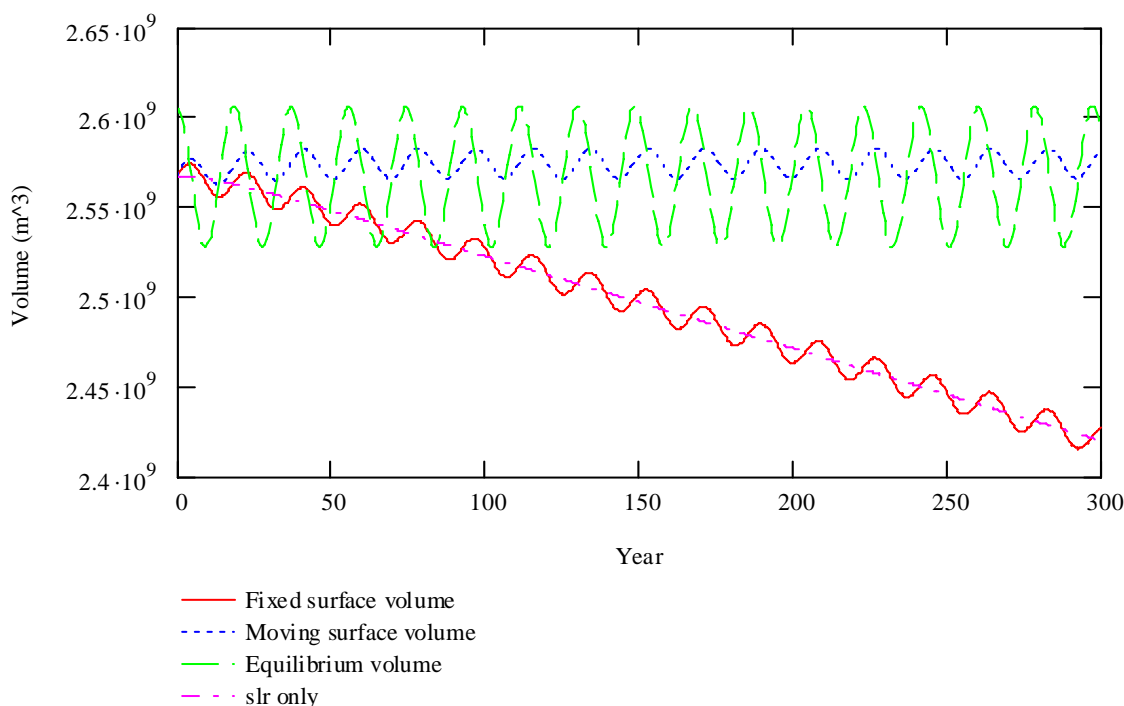


Figure 3. Schematic of elements for a tidal inlet as used in the ASMITA model

The variation in these volumes depends on the transport of sediment in and out of the elements and any changes to the water volume itself. The latter may be due to sea level rise, subsidence of the bed, or any form of progressive change in the basin volume. Hence, over the long-term (time scales much longer than a tidal cycle) the rate of change of the element volume depends on the residual flux, the change in sea level as follows and any change in tidal range:

$$\frac{dV_i}{dt} = \sum_j J_{i,j} + S_i \frac{d\zeta}{dt} + S_i \frac{d(tr)}{dt} \quad (7)$$

where J is the sediment volume flux between elements i and j , S is the plan area of the element and ζ is the elevation of mean sea level. The residual sediment flux between two elements is assumed to have advective and diffusive components:

$$J_{i,j} = q_{i,j} \cdot c_i - D_{i,j} A_{i,j} \frac{\partial c}{\partial x} \quad (8)$$

where q (m^3s^{-1}) is the residual discharge rate, D (m^2s^{-1}) is the diffusion coefficient between the two elements and A (m^2) is the vertical area of the interface between the two elements. This can be written in a form similar to the r.h.s. of equation 4:

$$J_{i,j} = q_{i,j} \cdot c_i - \delta_{i,j} \cdot (c_i - c_j) \quad (9)$$

In equations 8 and 9 the subscripts i and j refer to the elements the transport is from and to respectively. The advective component assumes that the concentration of the flux is that of the element supplying the sediment, which is important for external inputs such as from the rivers. In contrast the diffusive component simply uses the gradient between the elements.

The equations presented for the single element model can be conveniently written in matrix form to represent a multi-element model. Here we follow the naming convention of Kragtwijk *et al.* (2004) for volumes and add some additional terms to make the model more general with respect to diffusion, advection and the sources of perturbations in the volume.

Vectors are defined for a number of the variables as follows:

\underline{V}	element volumes;
\underline{S}	element surface areas;
\underline{c}_e	local equilibrium concentrations;
\underline{c}	element concentrations;
$\underline{\gamma}$	volume ratios (V_{ke}/V_k) ⁿ ;
$\underline{\sigma}$	surface area ratios (S_{ke}/S_k) ⁿ ;
$\underline{\delta}_E$	horizontal exchange coefficients with reality;
\underline{n}	concentration transport exponent, taken as positive for wet volumes and negative for sediment volumes.

In addition use is made of the following diagonal matrices:

W	vertical exchange coefficient w ;
S	surface areas;
I	unit or identity matrix;
M	unit matrix with sign of n ;
C_{ext}	matrix of concentrations for fluxes into the system from the environment.

The matrix **D** reflects the structure of the horizontal exchange between elements and with the outside world. For an n-element system with all elements linked and exchanging sediment with external environments, this would take the following form:

$$\mathbf{D} = \begin{pmatrix} \sum \delta_{1,n} + \delta_{1,E} & -\delta_{1,2} & \dots & -\delta_{1,n} \\ -\delta_{2,1} & \sum \delta_{2,n} + \delta_{2,E} & \dots & -\delta_{2,n} \\ \dots & \dots & \dots & \dots \\ -\delta_{n,1} & -\delta_{n,2} & \dots & \sum \delta_{n,n} + \delta_{n,E} \end{pmatrix} \quad (10)$$

For a 3-element system in which only element 3 is connected to the external environment (e.g. a tidal flat, channel and delta) this would take the form:

$$\mathbf{D} = \begin{pmatrix} \delta_{1,2} & -\delta_{1,2} & 0 \\ -\delta_{2,1} & \delta_{2,1} + \delta_{2,3} & -\delta_{2,3} \\ 0 & -\delta_{3,2} & \delta_{3,2} + \delta_{3,E} \end{pmatrix} \quad (11)$$

For diffusion the matrix is symmetric because $\delta_{i,j} = \delta_{j,i}$ and the direction of transport depends on the concentration gradient. However, for residual discharge the transport has a specified direction and as already noted we adopt the convention of transport from element *i* to element *j*. The matrix for an n-element system is then given by:

$$\mathbf{Q} = \begin{pmatrix} -\sum q_{1,n} + q_{1,E} & q_{1,2} & \dots & q_{1,n} \\ q_{2,1} & -\sum q_{2,n} + q_{2,E} & \dots & q_{2,n} \\ \dots & \dots & \dots & \dots \\ q_{n,1} & q_{n,2} & \dots & -\sum q_{n,n} + q_{n,E} \end{pmatrix} \quad (12)$$

Where $q_{n,E}$ all relate to fluxes out of the system. In order to ensure continuity of water mass we also require that, for each element, the sum of the discharges in and out of the element is zero, i.e. $\sum q_i = 0$.¹

The basic equations can now be written:

$$\frac{dV}{dt} = \mathbf{MW}(c_e - \underline{c})$$

$$(\mathbf{D} + \mathbf{Q}^T)\underline{c} = \mathbf{W}(c_e - \underline{c}) + \mathbf{C}_{ext}(\underline{\delta}_{ext} - \underline{q}_{ext}) \quad (13)$$

$$\underline{c}_e = c_E \underline{\gamma}$$

¹ However, provision is made for the sediment concentrations associated with these discharges to vary.

From which the following expression can be derived for the rate of change of volume (note the vector \underline{d} has the opposite sign to that given in equation 17 of the paper by Kragtwijk *et al.*, 2004):

$$\frac{dV}{dt} = \mathbf{B}\underline{\gamma} - \underline{d} + \underline{S} \frac{d\zeta}{dt} + \underline{S} \cdot \times \underline{trs} \frac{d(tr)}{dt} + \underline{\Delta V}$$

where (14)

$$\mathbf{B} = c_E \mathbf{MWS}(\mathbf{I} - (\mathbf{D} + \mathbf{Q}^T + \mathbf{WS})^{-1} \mathbf{WS})$$

$$\underline{d} = \mathbf{C}_{ext} \mathbf{MWS}(\mathbf{D} + \mathbf{Q}^T + \mathbf{WS})^{-1} (\underline{\delta}_{ext} - \underline{q}_{ext})$$

Here tr refers to the tidal range and the expression $\cdot \times \underline{trs}$ refers to element by element multiplication by $\frac{1}{2}$ with a sign that reflects whether the element is influenced by high or low water. The term $\underline{\Delta V}$ refers to any other changes in volume, introduced at any given time, within each element, e.g. due to reclamation or dredging.

Equilibrium and fixed volumes then have a similar form to equation 6, where δtr is the variation in the tidal range from its mean value:

$$\underline{V}_e = \underline{\alpha} \cdot P = \underline{V}_o + \underline{\alpha} \cdot \Delta P$$

$$\underline{V}_f = \underline{V}_m - \underline{S} \cdot \zeta - \underline{S} \cdot \times \underline{trs} \cdot \delta tr$$
(15)

The relationship between estuary volume and surface area

As estuaries vary throughout their length the depth change is generally much smaller than the width change. This obvious statement induces a corollary that the tidal, wave, fluvial and geotechnical effects that define an estuary geometry induce much larger changes to the width of a cross-section than to the depth. In addition, the downstream part of an estuary, which is predominantly tidal, tends to experience less proportional change in depth with distance than the head of an estuary which is much more affected by fluvial discharges. Since the volume and surface area within an estuary tends to be dominated by the much wider and deeper downstream reaches, where cross-section area correlates highly with width, estuary volume also tends to correlate highly with surface area.

Figure 3 indicates how in simple terms a change (δA) in channel cross-section area can be related to the change in width (δW), i.e.. $\delta A = h \cdot \delta W$. Because of the above arguments it is possible to extend this idea to surface area (S) and volume (V), i.e. $\delta V = h \cdot \delta S$. Moreover, the same argument can be made for change in tidal flat volume and surface area.

Variable surface area

For a single element (such as a channel) linked to the open sea, the above arguments lead to the following expression (in the absence of sea level rise and tidal node variation):

$$\frac{dS}{dt} = \frac{1}{h} \cdot \frac{dV_m}{dt} = \frac{1}{h} \cdot \frac{w \delta c_E S}{\delta + wS} \left[\left(\frac{V_e(t)}{V_m(t)} \right)^n - 1 \right] = \frac{\hat{w} \hat{\delta} c_E S}{\hat{\delta} + \hat{w} S} \left[\left(\frac{S_e(t)}{S_m(t)} \right)^n - 1 \right]$$
(16)

where:

$$\hat{q} = \frac{q}{h}; \quad \hat{\delta} = \frac{\delta}{h} \quad \text{and} \quad \hat{w} = \frac{w}{h} \quad (17)$$

The effect of sea level rise and nodal tide variation is included by considering the change in surface area that arises from a change in mean sea level and a change in tidal range. The non-linear equations (for the single element model) describing the variation in surface area are therefore similar to equation 5 and 6 for volumes, viz:

$$S_e(t) = \hat{\alpha} \cdot \frac{P}{tr} = S_o + \hat{\alpha} \cdot \frac{\Delta P}{tr}$$

$$\frac{dS_m}{dt} = \frac{\hat{w} \hat{\delta} c_E S}{\hat{\delta} + \hat{w} S} \left[\left(\frac{S_e(t)}{S_m(t)} \right)^2 - 1 \right] + \frac{d\zeta}{dt} \cdot R + i\omega \frac{\eta}{2} e^{i\omega t} \cdot R \quad (18)$$

$$S_f(t) = S_m(t) - \frac{d\zeta}{dt} \cdot R \cdot t - \frac{\eta}{2} e^{i\omega t} \cdot R$$

The parameter R represents the change in area for a unit change in water level and is given by $R = n L m_b$, where n is the number of bed surfaces in the element (generally 2 for the two sides of the estuary), m_b is the transverse bed slope (i.e., slope is given by the ratio 1: m_b) and L is the length of the element.

As for volumes, this can be applied to multiple elements and the rate of change for the surface area \mathbf{S} , is thus given by:

$$\frac{d\mathbf{S}}{dt} = \mathbf{F} \underline{\sigma} - \underline{e} + \mathbf{R} \frac{d\zeta}{dt} + \mathbf{R} \cdot \times \mathbf{trs} \frac{d(tr)}{dt} + \underline{\Delta S}$$

where (19)

$$\mathbf{F} = c_E \mathbf{M} \hat{\mathbf{W}} \mathbf{S} (\mathbf{I} - (\hat{\mathbf{D}} + \hat{\mathbf{Q}}^T + \hat{\mathbf{W}} \mathbf{S})^{-1} \hat{\mathbf{W}} \mathbf{S})$$

$$\underline{e} = \mathbf{C}_{ext} \mathbf{M} \hat{\mathbf{W}} \mathbf{S} (\hat{\mathbf{D}} + \hat{\mathbf{Q}}^T + \hat{\mathbf{W}} \mathbf{S})^{-1} \cdot (\hat{\delta}_{ext} - \hat{q}_{ext})$$

where $\hat{\mathbf{D}}$, $\hat{\delta}_E$ and $\hat{\mathbf{W}}$ are derived from \mathbf{D} , δ_E and \mathbf{W} in accordance with equation 17. The equilibrium and fixed surface areas then have a similar form to equation 15:

$$\underline{S}_e = \hat{\alpha} \cdot P = \underline{S}_o + \hat{\alpha} \cdot \Delta P$$

$$\underline{S}_f = \underline{S}_m - \mathbf{R} \cdot \zeta - \mathbf{R} \cdot \times \mathbf{trs} \cdot \delta tr \quad (20)$$

Tidal prism relationships (including example analysis of the Humber Estuary)

Equation 1 postulates a link between the equilibrium volume and the tidal prism. Similarly the value of the equilibrium surface area S_e in Equation 17 is likely to vary with the tidal prism. Townend (2005) showed that the tidal prism exhibits a strong correlation with the plan area and volume of the estuary. From a sample of 66 UK estuaries he found that the total surface area, total volume and tidal prism exhibited the following relationships:

$$\begin{aligned}
 S_{mtl} &= 0.42P^{0.96} & R^2 &= 0.92; \\
 V_{mtl} &= 0.073P^{1.13}, & R^2 &= 0.92.
 \end{aligned}
 \tag{21}$$

The elements used within ASMITA comprise deltas, channels and tidal flats. The UK estuaries database suggests that the channels and flats (when represented as a single element) can be adequately represented by simple linear relationships with exponents of one:

Table 1. Linear form-prism ratios

	Volume	R²	Surface Area	R²
Channel	$V_c = 0.418P$	0.93	$S_c = 0.076P$	0.95
Tidal flats	$V_f = 0.163P$	0.74	$S_f = 0.077P$	0.47
			$S_f = 28.9P^{0.75}$	0.82

However, individual estuaries can have values significantly different from the above. The following are the average values for the Humber over the period 1851-2000 (based on 22 bathymetric data sets) with standard deviations indicating a small variation over time:

Table 2. Linear form-prism ratios for the Humber Estuary

	Volume	St.Dev	Surface Area	St.Dev
Channel	$V_c = 0.738P$	0.021	$S_c = 0.130P$	0.003
Tidal flats	$V_f = 0.249P$	0.011	$S_f = 0.067P$	0.003

One of the strongest correlations revealed by the UK data is between the surface area at mean tide level, prism and tidal range, which takes the form:

$$S_{mtl} = 1.07 \frac{P}{tr} \quad (R^2 = 0.996)
 \tag{22}$$

This implies that the tidal prism is essentially the plan area at mean tide multiplied by the tidal range (particularly for the larger estuaries with $P > 10^7 m^3$) and that the cross-shore shape of the intertidal is of secondary importance. Although the regression is strong it must be recognised that the error in predicting the plan area of an individual estuary can still be $\pm 30\%$ based on this data set, with most of the data points falling below this forced regression line (i.e. exponent=1 and intercept=0). The surface area at low water is also reasonably well represented by this form of relationship but the area of the tidal flats shows a poorer correlation, Table 3. The coefficients specific to the Humber are similar for the mean tide and channel (MLW) relationships but differs significantly for the tidal flats.

Table 3. Linear form-prism ratios for surface area as a function of P/tr

	UK Estuaries	R²	Humber Estuary
Channel	$S_c = 0.775P/tr$	0.97	$S_c = 0.733P/tr$
Tidal flats	$S_f = 0.945P/tr$	0.77	$S_f = 0.386P/tr$
Mean tide level	$S_{mtl} = 1.07P/tr$	0.99	$S_f = 0.96P/tr$

Hypsometry relationships

An alternative approach is to use the equilibrium hypsometry relationships for channels (Cao and Knight, 1997) and tidal flats (Friedrichs and Aubrey, 1996) to define the form of the estuary cross-section. The resulting relationships can then be written in terms of volume and surface area by considering a representative length. Some trials for a number of estuaries suggest that this form description provides a reasonable characterisation of estuary cross-sections.

Friedrichs and Aubrey (1996) show that the lower intertidal width is a function of the equilibrium flow velocity and the angular tidal frequency: $(w_o - w_{lw}) = u_{eq}/\omega$. This implies that the lower intertidal width reflects flow field, the erodibility of the sediments and the rate of convergence of the estuary. We therefore assume that the slope of the lower intertidal is a characteristic value, which in terms of surface area is given by:

$$m_{eq} = \frac{S_o - S_{lw}}{2\eta} \quad (23)$$

Using the initial estuary surface areas at mtl and lw, with the tidal range, the equilibrium slope can be calculated using equation 23. Similarly, the d-n ratio = $(d-\eta)/(n+1)$ can be obtained as the ratio of the volume and area at low water. If the slope and d-n ratio are treated as being constant, equations 24 can be used to derive updated equilibrium volumes and areas for the channels and flats given a new value of the tidal prism.

$$\begin{aligned} S_{lw} &= \frac{1}{2\eta} (V_p - (1 + \pi) \cdot \eta^2 m_{eq}) & V_{lw} &= \frac{d - \eta}{n + 1} \cdot S_{lw} \\ S_{fl} &= \left(1 + \frac{\pi}{2}\right) \cdot 2\eta \cdot m_{eq} & V_{fl} &= (1 + \pi) \eta^2 m_{eq} \end{aligned} \quad (24)$$

Fitting equation 24 to the UK estuaries database produces reasonable fits to the data. In particular, V_{lw} is well represented and the fit lines go through the centre of the data, whereas for V_{fl} , and S_{fl} the fitted lines represent lower bounds.

Basis of equilibrium volumes and areas in ASMITA

The ASMITA model, as presented, makes use of the general nature of the results described above and provides the user a number of options.

(i) Equilibrium based on linear tidal prism functions:

This method assumes that the governing equation for equilibrium volume is a linear function of the tidal prism and for surface area is a linear function of prism/tidal range. The user is able to set the values of the proportionality coefficients separately for each element. In the code the default values for these coefficients are 1 and if these are used, the routine calculates the coefficients from the initial values of volume, surface area, tidal prism and tidal range for each element. Thereafter, the coefficients are treated as constants. (NB: This uses the individual element volume or surface area as a proportion of the total tidal prism. This implicitly scales the coefficient to the size of the element relative to the total tidal prism. When considering internal perturbations to the system, e.g. due to reclamation or dredging, it may be more appropriate to use the prism from each element to the tidal limit. This option has not however been implemented.)

(ii) Equilibrium based on hypsometry functions:

This method uses the initial values of volume, surface area, tidal prism and tidal range to calculate the equilibrium slope and the d-n ratio, as defined above. These values are then treated as constants and the equations for the equilibrium hypsometry (24) are used to compute volumes and areas for the channels and flats, taking account of change in tidal range and tidal prism. The method is activated by setting the volume equilibrium coefficients for every element to zero.

References

di Silvio G, 1989, Modelling the morphological evolution of tidal lagoons and their equilibrium configurations, In: 22nd IAHR Congress, IAHR, pp. C-169-C-175.

Cao S, Knight D, 1997, Entropy-based design approach of threshold alluvial channels, *Journal of Hydraulic Research*, 35(4), 505-524.

Eysink WD, 1990, Morphological response of tidal basins to change, In: 22nd International Conference on Coastal Engineering, ASCE, Washington, 2, pp. 1948-1961.

Friedrichs CT, Aubrey DG, 1996, Uniform bottom shear stress and equilibrium hypsometry of intertidal flats, In: Pattiaratchi C (Ed.), *Mixing in estuaries and coastal seas*, American Geophysical Union, Washington, pp. 405-429.

Jeuken M.C.J.L.; Wang, Z.B.; Keiller, D.; Townend, I.; Liek, G.A.; *Morphological response of estuaries to nodal tide variation*, In: Proc. Int. Conf. on Estuaries and Coasts 2003 (IRTCEs), Zhejiang Institute of Hydraulics and Estuary, Hangzhou, China, November 9-11, 2003.

Kragtwijk NG, Stive MJF, Wang ZB, Zitman TJ, 2004, Morphological response of tidal basins to human interventions, *Coastal Engineering*, 51, 207-221.

Niedoroda AW, Reed CW, Das H, Kock J, Donoghue J, Wang ZB, Stive MJF, 2003, Modelling large-scale morphodynamics of complex coastal systems, In: *Coastal Sediments '03: Crossing Disciplinary Boundaries - Proceedings of the Fifth International Symposium on Coastal Engineering and Science of Coastal Sediment Processes, East Meets West Productions (EMW) Inc., Corpus Christi, USA*, pp. 1-14.

Stive MJF, Capobianco M, Wang ZB, Ruol P, Buijsman MC, 1998, Morphodynamics of a tidal lagoon and adjacent coast, In: *Physics of Estuaries and Coastal Seas: 8th International Biennial Conference on Physics of Estuaries and Coastal Seas, 1996*, A A Balkema, Rotterdam, pp. 397-407.

Townend IH, 2005, An examination of empirical stability relationships for UK estuaries, *Journal of Coastal Research*, 21(5), 1042-1053.

van Dongeren AR, de Vriend HJ, 1994, A model of morphological behaviour of tidal basins, *Coastal Engineering*, 22, 287-310.

van Goor MA, Stive MJE, Wang ZB, Zitman TJ, 2001, Influence of relative sea level rise on coastal inlets and tidal basins, In: Hanson H, Larson M (Eds.), *Coastal Dynamics '01*, American Society of Civil Engineers, Reston, Virginia, pp. 242-251.

van Goor MA, Zitman TJ, Wang ZB, Stive MJF, 2003, Impact of sea-level rise on the morphological equilibrium state of tidal inlets, *Marine Geology*, 202, 211-227.

Wang ZB, Karssen B, Fokkink RJ, Langerak A, 1998, A dynamic-empirical model for estuarine morphology, In: Dronkers J, Scheffers MBAM (Eds.), Physics of Estuaries and Coastal Seas, Balkema, Rotterdam, pp. 279-286.

Wang ZB, Langerak A, Fokkink RJ, 1999, Simulation of long-term morphological development in the Western Scheldt, In: IAHR Symposium on River, Coastal and Estuarine Morphodynamics, University of Genova, Department of Environmental Engineering, II, pp. 367-376.

Wang, ZB, de Vries, M, Roelfzema, A, Kenny, A and Goss-Custard, J, 2000, ESTMORPH Humber Estuary Model and Habitat Evaluation; Modelling of long-term morphological development of Humber Estuary plus ecological assessments. In EMPHASYS Consortium (eds.) Modelling Estuary Form and Process, Final Report, EMPHASYS Estuaries Research Programme Phase 1, MAFF Contract CSA 4938, HR Wallingford, TR111, pp.157-171.