COUPLED HYDRAULIC AND ENTROPY RELATIONSHIPS

Method Indicator		
Bottom-Up	Hybrid	Top-Down
	YES	

Summary of key issues

Issue	**Description**
Description	The technique explores the conditions that are likely to prevail close to dynamic equilibrium for open systems with flows of matter and energy. It also has the potential to be used to examine system behaviour.
Temporal Applicability	Typically applied over a scale of 10 to 100 years (short-term).
Spatial Applicability	Typically can be used throughout the whole estuary.
Links with Other Tools	Can be used with Regime relationships, Accommodation space, expert geomorphological analysis, behaviour models, for example.
Data Sources	Hydraulic depth High water levels Low water levels Tidal prism Volume at LW Surface area Cross sectional area at mouth
Necessary Software Tools / Skills	Necessary software for modelling hydrodynamics such as Mike 11. Skills include a good understanding of the theory of Energy/Entropy as well as an understanding of modelling and geomorphology.
Typical Analyses	To look for perturbations from the most probable state, both in the existing regime and with any imposed changes (developments, sea level rise, etc).
Limitations	The main use is as a diagnostic tool. The method has not yet been developed in a way that allows it to be used as a predictive tool.
Example Applications	Humber Estuary

Introduction

The theoretical concept of minimum entropy production was first proposed by Prigogine (1955) and is excellently explained in general terms by Atkins (1984). The concept was applied to rivers by Leopold and Langbein (1962), who argued that the entropy production for the system, as a whole, should be a minimum. They combined this argument with conventional continuity, friction and sediment transport relationships and uniform energy per unit mass and uniform stream power to derive discharge relationships for the hydraulic geometry of a river, as summarised in Regime relationships. They noted that uniform energy would lead to a straight basin profile and minimum total work (entropy production) would give rise to a concave basin profile.

These concepts have been re-interpreted for the case of a bi-directional flow in an estuary and applied to a number of UK systems (Dun & Townend, 1998; Townend, 1999). The model is able to highlight the difference between systems, where some show a clear exponential upstream decay, whereas others show a more linear variation in the rate of energy dissipation. This reflects the interplay between minimum entropy production for the system as a whole and the fact that uniform work is but one way of achieving this most probable state, as identified by Leopold and Langbein (1962). Translating this into identifying the physical controls that determine the relative balance of this interplay is an aspect that still needs further research.

Where multiple bathymetries over time are available the *E*st*E*nt model can be used as a diagnostic tool (Townend & Dun, 2000) and it was examined for a range of different estuaries (Emphasys, 2000). Future developments are likely to explore a more complete theoretical development and the application in some form of iterative mode with a suitable hydraulic model (possibly using annealing techniques as mechanism for exploring a range of possible states to find the most probable).

Entropy-based relationships are a method of characterising the most probable hydraulic state. Their use within Phase 1 of the Estuaries Research Programme has been developed previously by Dun and Townend (1998) and Gill (2000). Whilst new work as part of this project has clarified the potential for this approach, it has also highlighted the fact that much more needs to be done before this can be presented as a useful tool. For the purposes of this report, attention is therefore confined to the more established concepts and their use.

Overview of technique

The existing basis for applying entropy to estuaries is confined to considering energy flux variation along the length of the estuary.

General concepts and definitions

Although predominantly used in thermodynamics, statistical mechanics and information theory, there are many interpretations, as expressed by Williams (1997), who presented the information in Table 1 to illustrate what constitutes high and low entropy. Even within the confines of thermodynamics, there is often a perceived difficulty because of the close inter-relationship with other thermodynamic properties and the confusion arising from the interpretation of different forms of energy, notably internal energy. A very clear explanation is provided in the two books by Atkins (1984; 2003).

Table 1. Extreme or opposite states of entropy (after Williams, 1997)

	High Entropy	**Low Entropy**
Proportion of energy available to do work	Low	High
Degree of order, sorting or separation	Low	High
Probability of a selected event	Low	High
Probability of events	Equally probable	Preordained outcomes
Type of distribution	Uniform	Highly uneven
Degree of certainty	Low	High
Predictability	Low, random and unpredictable	High leading to accurate forecasts
Range of outcomes	Wide variety	Constrained
Amount of diversity	Large	Small
Degree of surprise	Lots	Little or none
Amount of information	High	Low
Information needed to specify system state	Large	Small
Accuracy of data	High	Low

The energy of a system is the sum of the kinetic and potential energies of all the particles. However for many particle bodies (i.e. matter) the energy can have the additional property of structure or order, which is referred to as coherence. Thus incoherent motion is the random thermal motion of particles. Work stimulates coherent motion on the system or its surroundings. For this, kinetic and potential energy are the main stores of coherent particle motion and the act of doing work causes a reduction in coherence and a transfer of heat, which stimulates incoherent thermal motion. Where some structure resides, such that there is a temperature gradient, the system retains a more limited capacity to do work.

Every system is associated with energy and entropy. When matter undergoes a transformation from one state to another, the total energy remains the same but the total entropy can only increase. Energy stored at a high temperature has a low entropy and a high quality (where quality is a measure of the energies potential to do work). Entropy describes the way energy is stored. The natural direction of change causes a decline in energy quality (i.e. its availability to do useful work reduces). Systems always evolve in time in such a way that the total entropy of the system and the environment increases. If a system starts in a highly organised and, therefore, highly improbable state (with a high degree of coherence), the spontaneous motion of the particles causes the system to reach other states and hence find more probable states. As it does so, it is unlikely to return to the less probable state because the random particle motions have only a remote possibility of taking it there by chance. Within the universe, energy can be dispersed in many ways and consequently it progresses towards equilibrium almost irreversibly.

Jørgensen & Svirezhev (2004) also define the *Supersystem* to describe the system and its environment. For an isolated system, thermodynamic equilibrium is a time-invariant state in which there are no further physical or chemical changes in the system. The system evolves towards this state because of irreversible processes. It therefore follows that when these processes cease to operate, the system has reached a state of equilibrium. For two or more systems which exchange energy or matter, it follows that equilibrium exists when all the systems have reached a uniform temperature.

For a system with constant volume and temperature, the rate of entropy production is equal to the rate of change of free energy (Helmholtz energy), which is in turn equivalent to energy dissipation (for the system) and these quantities will be minimal. For an open system, this will also depend on the exchanges of free energy and entropy between the system and the environment.

Hydro-geomorphological context

With hydrodynamic systems, the useful energy is in the relative elevation and movement of water and sediment due to run-off, tides and density flows. These movements are relatively ordered and the flow of energy can be well defined. However there are many processes that act to break down this structure and increase the entropy of the system.

A strong current with ordered kinetic and gravitational potential energy may have eddies that form when it passes a change in bathymetry or opposes another current. These eddies will break off from the main flow and get broken down into smaller turbulent flows until the energy is eventually dissipated through viscosity (friction). The useful energy of the current (coherent motion) has broken down to random thermal motion of the water particles (incoherent motion). Other dissipating processes include friction with the seabed, wave breaking, sediment transport and processes associated with gradients in temperature, density, etc.

Riverine or estuarine energy comprises primarily kinetic and potential energy, although heat and chemical exchanges will also result in energy flows. This kinetic and potential energy represents coherent motion, because the energy can be used to do work. By contrast, heat stimulates incoherent motion, which is random, chaotic and uncorrelated. The first law of thermodynamics requires that energy is neither created nor destroyed, giving rise to the conservation of energy such that the energy supplied to a system by heating, doing work (where heat and work are forms of transfer and not forms of energy) or mass fluxes must equal the increase in energy of the system (comprising internal, kinetic and potential energies). The second law of thermodynamics then describes the way in which energy disperses, such that natural processes are accompanied by an increase in the entropy of the universe. Entropy represents the degree of coherence to the particle motion and the natural direction is one that causes this coherence to decline. Low entropy therefore reflects highly coherent energy, whereas high entropy relatively incoherent thermal motion.

In an open system, such as a river or estuary, we are concerned with the flow of matter through the system, in addition to the dispersion of energy. Such flows have, at least local structure, which exists for as long as there is a flow of matter, or energy is being dispersed. This structure may be regarded as synonymous with coherence and clearly can occur in both space and time. Given that this form of structure serves to disperse energy, it is sometimes referred to as a dissipative structure. The Second Law tells us that such structures cannot emerge spontaneously out of disorder. We can however generate local structure or order, providing there is a greater increase in disorder elsewhere.

An estuary/river system is viewed as a dissipative structure, in which coherent forms of energy are dispersed to more incoherent forms, leading ultimately to a transfer of kinetic and potential energy to the thermal motion of the sea. This simple structure is of course complicated by the action of the tides within both the sea and estuary, and other flows, such as those induced by temperature and salinity gradients.

Previous Applications

As far as estuaries are concerned the principle application of this approach is by Dunn and Townend (1999), Townend and Dunn (2000), following the approach proposed by Leopold and Langbein (1962) almost 40 years earlier for rivers. However, additionally, there is an extensive body of literature covering a wide range of applications to different aspects of river basin (Scheidegger, 1961; Rodríguez-Iturbe & Rinaldo, 1997), rivers (Brebner & Wilson, 1967; Yang, 1971; Yalin & Ferreira da Silva, 2001), and beach morphology (Dean, 1977; Bodge, 1992; Lee & Mehta, 1997). These papers have all adopted an approach that has a thermodynamic basis. There is a further, extensive literature covering a wide range of applications that use a statistical approach based on the Principle of Maximum Entropy, or POME, some of which are cited above for hydraulic geometry. Both methods are briefly reviewed below.

The concepts of minimum entropy production, unit stream power and more generally the extremal hypothesis have been subject to extensive criticism over the last 30 years (Davy and Davies, 1979; Griffiths, 1984; Lamberti, 1988 and Yang, 1971). Much of this comes about either because of a lack of appreciation of the underlying assumptions in the thermodynamic derivation, particularly with regard to the work on minimum entropy production by Prigogine, or because extremal methods are being applied without a broader physically based context. To overcome this and establish a valid critique of the approach, a broader context is required. A qualitative assessment is provided in the review that follows but, in order to develop the method further, this will need to be supported by a more comprehensive derivation of the governing equations making use of established hydrodynamic and thermodynamic principles. This work is underway but is not yet at a stage where it can be usefully reported.

Thermodynamic approach

Early work by Langbein (1963) and Myrick and Leopold (1963) used the principles of the Leopold and Langbein paper to develop regime relationships for velocity, width, depth and slope. They do not however revisit the concept of entropy production in the context of a system with tidal flows (bi-directional), where the discharge is no longer independent of the system morphology (width and depth).

For the estuary case, the discharge varies along the length of the system. This variation will be dependent on longitudinal changes to channel depths and widths, along with the amount of water storage within a particular reach and the dissipative action of bed friction. Thus, the discharge at a particular point along an estuary is dependent on the channel morphology and frictional losses. This is a fundamental distinction from the fluvial case, in which discharge rates are independent of the channel shape but are, rather, dependent on the rainfall and catchment characteristics.

In an attempt to incorporate this interdependence, Dunn and Townend (1998); Townend (1999) followed the line of argument proposed by Leopold and Langbein (1962) but applied to the energy flux rather than the energy head. This led to a similar equation but retaining the discharge:

$$\left(\frac{dS\,/dt}{Q}\right)=\frac{d\,(HQ)/\,dx}{HQ} \qquad (1)$$

Where H is the specific energy head. This can be solved to give an expression of the form:

$$\ln HQ = Cx + D \qquad (2)$$

Statistical Approach

The concepts of information entropy developed by (Shannon, 1948) and the principle that an equilibrium system under steady constraints tends to maximise the entropy, proposed by (Jaynes, 1957), have been extensively used to suggest most probable distributions[1] of a range of parameters studied in open channel flow.

The general approach is to define a probability density function for the chosen parameter, which can be used to define the entropy in terms of probability (Shannon, 1948). Jaynes (1957) has shown that an equilibrium system under steady constraints tends to maximise its entropy. This is commonly known as the Principle of Maximum Entropy (POME). In some cases this principle is used directly to examine the basis of equal probability (e.g. Singh et al. 2003) and in others the method is used in conjunction with the calculus of variations to derive a solution (e.g. Cao and Knight, 1996).

The difficulty seems to be that in taking a broader view one needs a basis for determining which parameters probabilities can be maximised based on the constraints imposed on the system. It would therefore seem that a complete solution of the regime problem is more likely to be arrived at through the application of thermodynamic entropy, based on physical processes, rather than statistical entropy. This is not to deny the valuable role of the statistical method for analysing and interpreting measured data. The issue however is that a

[1] In this context, the most probable distribution refers to the system state that is most likely

rational framework is needed, so that the choice of which parameters to maximise is a function of the physics and the boundary conditions, rather than some arbitrary choice.

Summary of applications

The early applications of entropy concepts considered either maximising the entropy or minimising the entropy production (although the determination of entropy production should often more accurately be termed entropy flux). This made use of an analogy between thermodynamic temperature and land surface elevation. A number of workers subsequently sidestepped the need to make use of this analogy and developed the concept of minimum rate of energy dissipation. A special case, when other forms of energy dissipation are ignored (e.g. any dissipation due to the sediment load) and only the water discharge is considered, leads to the minimisation of the stream power (QS_e). Further, by ignoring any slope or velocity variations across the channel leads to the minimisation of the unit stream power (vS_e).

Alternatively, assuming that the energy slope, S_e, is constant locally in a reach or meander, and using thermodynamic arguments relating to the earth and its sub-compartments down to the scale of river basins, rivers should minimise the average channel velocity. This is entirely consistent with the more general finding that for thermodynamic systems operating in the linear or quasi-linear region, close to equilibrium, the forces and fluxes will adjust so that all unconstrained forces disappear and those that are constrained adjust to minimise the flux and hence the entropy production (or equivalently at dynamic equilibrium the entropy flux).

A third approach has been a statistical one, making use of the principle of maximum entropy (POME). This seeks to identify the most probable distribution of a variable by maximising the system entropy. This is done in terms of a probability density function for the chosen parameter, which can be used to define the entropy in terms of probability. The concept has been applied to a single variable such as slope, velocity, stream power, etc to determine its distribution. The principle has also been used to examine the relationship between the various hydraulic geometry parameters by identifying the proportion of adjustment attributable to each parameter and then using POME arguments to equate these under chosen constraints.

Given that landscapes are open systems subject to flows of matter and energy, there is a reasonable basis for expecting the equations of hydrodynamics and thermodynamics to provide a description of their behaviour. The physical arguments this presents will have greater explanatory power than simply adopting probabilistic arguments. That is not to say that such methods are excluded. Techniques such as the Principle of Maximum Entropy (POME) may well play a supporting role but the foundation should be defined using physical arguments and the various balance equations.

Approaches using such arguments can be derived from the basic entropy balance equation, and have been used in various guises in a wide range of applications, covering landscapes, rivers, estuaries and beaches. Reviewing the applications across a range of geomorphological units has revealed a number of inconsistencies that need to be addressed. In particular:

- The methods used should have a clear relationship with the fundamental equations of hydraulics and thermodynamics, so that simplifying assumptions can be clearly identified and their consequences understood.

- There needs to be a more careful handling of constraints and in particular accommodation of vertical and horizontal constraints.
- The specification of the minimum condition needs to be developed in a way that can be applied in a consistent manner across the different applications.

Interpretation for hydro-geomorphological systems

Estuary

The channels, meanders and storage areas in an estuary serve a similar purpose, as a means of adjusting the rates of energy storage and dissipation, as in the rivers case. In addition the intertidal functions both as a storage area to accommodate the discharge variations that occur on every tide and also to act as a means of adjusting the margins of the estuary to varying levels of wind-wave energy. This is typically much less than the energy inputs on the open coast but the mechanisms are similar. In the case of an estuary or less exposed open coast areas, this can be supplemented by saltmarshes, which further enhance the dissipative capacity of the system and serve to store a greater volume of sediment.

Application to estuaries

Leopold and Langbein (1962) applied the concept of minimum entropy production to the problem of river hydraulics and morphology. A subsequent paper by Langbein (1963) considered the application of the same approach to shallow estuaries. This however deals with an 'ideal' estuary (Pillsbury, 1956) and therefore is constrained by the assumption that the amplitude of the tidal elevation and velocity are constant throughout the system. The influence of the frictional terms (Lamb, 1932; Dronkers, 1986) and the interaction of M_2 and M_4 tidal constituents, referred to as the overtide, (Friedrichs & Aubrey, 1988) further limit the validity of Langbein's application of this approach to the case of an estuary.

In order to develop a more rigorous approach, the derivation of minimum entropy production in a river system was re-examined. This was found to be a special case of the more general case of a reach with bi-directional and variable discharge. The generalised formulation applies to the estuary case and can be used to investigate the relationship between morphology and tidal energy distribution. For the evolution to a probable state in a system near to equilibrium, it has been suggested that the entropy production per unit volume will tend to evolve to a minimum compatible with the conditions imposed on the system (Prigogine, 1955). Relating this to an estuary suggests that, in the long-term, a natural system will tend to evolve in an attempt to achieve the most probable distribution of tidal energy. However, the time taken to evolve to this state will be dependant on constraints imposed upon the system (such as geological constraints and supply of sediments). Such constraints may be significant enough to prevent the evolution to the most probable state in which entropy is maximised, or may induce a switch to some other steady state. Another complication is that the energy available to the system varies temporally over the evolutionary timescale, due to climatic changes, sea level rise, etc.

The concept of minimum entropy production per unit discharge has been derived for the more general case of a bi-directional variable discharge along a channel reach (Townend, 1999). Following a similar line of argument to that of Leopold and Langbein (1962), the longitudinal energy distribution along an estuary may be represented as:

$$\frac{1}{HQ}\frac{d(HQ)}{dx} = C_t \qquad (3)$$

where; C_t is a constant at time *t*, *H* is the specific energy head and *Q* is the discharge. This describes the energy distribution at any given stage in the tidal cycle. Considering the complete tidal cycle we can write:

$$\int HQ\,dt = e^{C^1 x + D^1} \qquad (4)$$

where: $\int HQ\,dt$ is the sum of the energy flux through a section at a distance x from the mouth of the estuary over a complete tide, or power, P(x). C^1 and D^1 are constants. This suggests that, for the most probable distribution of energy throughout an estuary (and thus a constant production of entropy per unit discharge), the energy transferred due to the tidal wave will decay exponentially in the upstream direction. This general model of variable discharge along a reach can incorporate energy introduced at the up-stream limits of the estuary as a result of river inputs. In order to generate a solution for Equation (4)**,** boundary conditions have to be defined in a similar manner to the fluvial case. Although it is possible to generate a tidal curve at the mouth of an estuary, it is not possible to generate a discharge curve, as this is dependent on the morphology of the estuarine channel. However, given an initial bathymetry, an appropriate analytical or numerical model can be used to generate the discharge curve via the solution of the equations of continuity and momentum.

The most probable distribution of energy can then be obtained using Equation (4), by noting that:

- At x = 0, C = P(0)
- At x = L, $D = \frac{1}{L}\ln\left(\frac{P(L)}{P(0)}\right)$

Using the values of P(0) and P(L) taken at the inner and outer boundaries, where L denotes the total length of the system, the theoretical distribution can be compared to the actual (numerical model or measured) energy flux distribution along the estuary's length. By comparing the most probable with the modelled results, areas that are likely to experience a loss of conveyance, can be identified. Depth and width changes may then be introduced into the model and the results compared with the updated most probable state solution. Examining the differences from the most probable provides a basis for assessing the relative direction of change from the most probable state (i.e. whether the system is getting closer to, or moving further away from, the preferred state); (Figure 1). The top plot shows the variation of the energy flux along the length of the estuary and is compared with the most probable distribution obtained using Equation (4). In the middle plot this is shown as the difference between the two curves in the upper plot. Finally, the lower plot shows the difference of a percentage of the actual value at any given distance along the estuary. Given the very large gradient in energy flux, along the length of the estuary, this provides a better indication of the deviation from the most probable state.

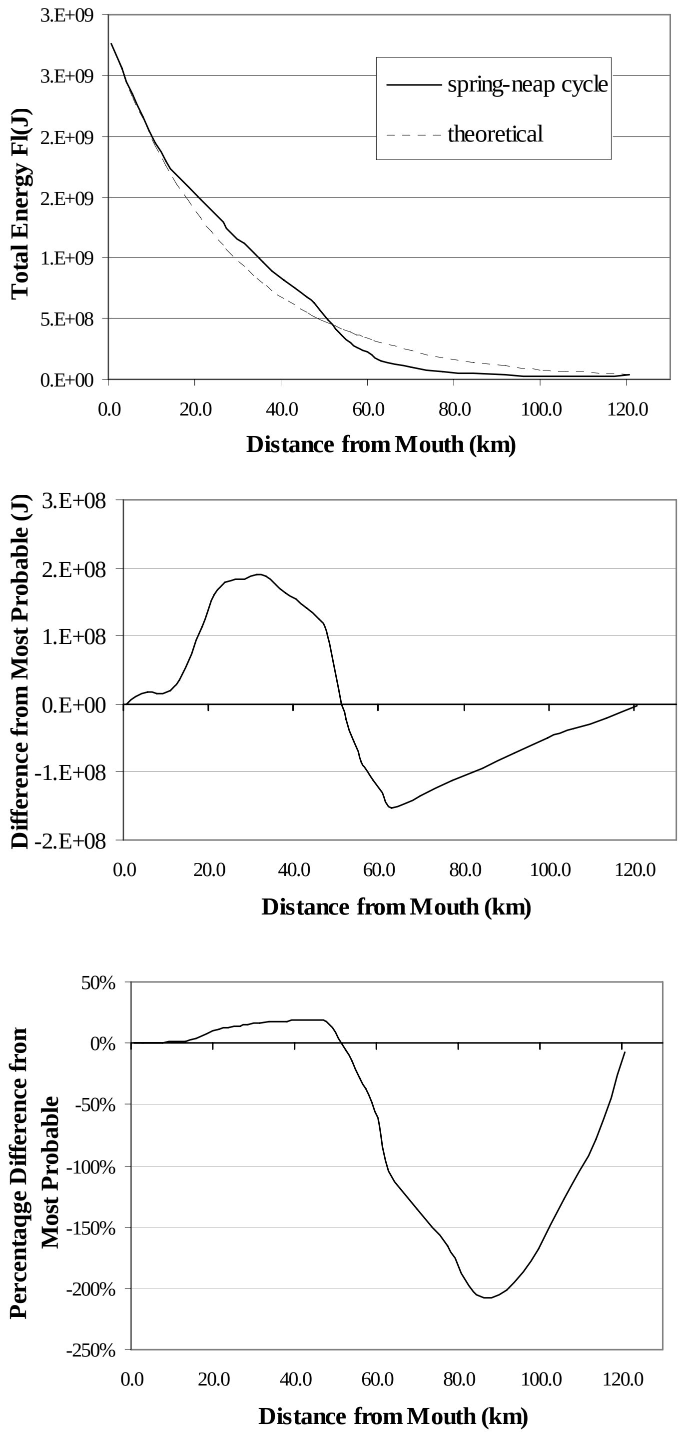

Figure 1. Variation in energy flux, and difference from most probable distribution

Influence of boundary conditions

In an estuarine system, defining just where to position the boundaries is not always obvious. Where the estuary rapidly opens to the sea it is usually possible to define a mouth. However, this may not be the system's boundary, which may extend to include the ebb tidal delta, or may now be located within an embayment. Similarly, at the upstream end, the tidal limit is an obvious choice but other constraints may dictate other intermediate boundaries.

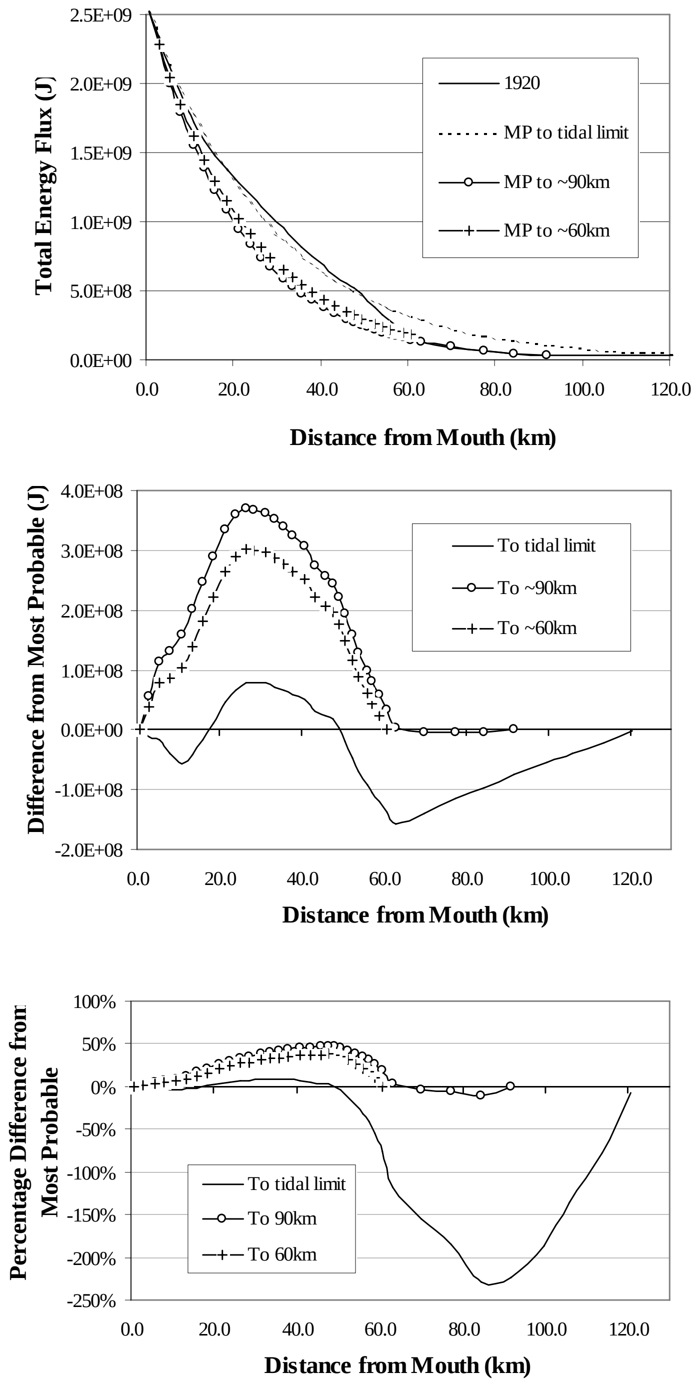

Figure 2. Influence of boundaries on most probable distribution

The effect of moving the boundary can be quite significant because, in the approach presented above; this is the basis for calculating the values of the constants C and D. This is illustrated in Figure 2, which shows the case presented in Figure 1, with inner boundaries for the definition of C and D at the 60, 90 and 120km (the tidal limit).

In the upper plot of total energy flux, the actual data are seen to accord well with the tidal limit boundary values, at least in the outer estuary. In contrast, the other limits, which only cover the outer estuary, exhibit a greater discrepancy. This is highlighted in the middle plot, which shows the absolute difference from the most probable values. When this is considered as a relative difference (lower plot) this, in effect, rotates the curve about the zero chainage point. The minimum entropy production concept is applicable within any given reach, as well as for the system as a whole. Thus it is important to note the context of the length being examined, as some part of the system as a whole, but it remains valid to consider the relative change and, in particular, the direction of change, within any given length.

Intermediate constraints

Internal constraints can also influence the form of the most probable state. This is an essential aspect of the derivation (Prigogine, 1955) and has recently been shown to have a determining influence in the resultant form of river regime equations (Singh *et al.* 2003). The point is nicely illustrated using data from the Bristol Channel (Dun, pers comm.). Using constraints based on the inner and outer boundaries the resultant theoretical curve is shown in Figure 3 (labelled “Boundary fit”). A simple exponential curve has also been fitted to the data (labelled “Best fit”). The difference of the data from the Best fit can be interpreted as the excess entropy production in the existing system.

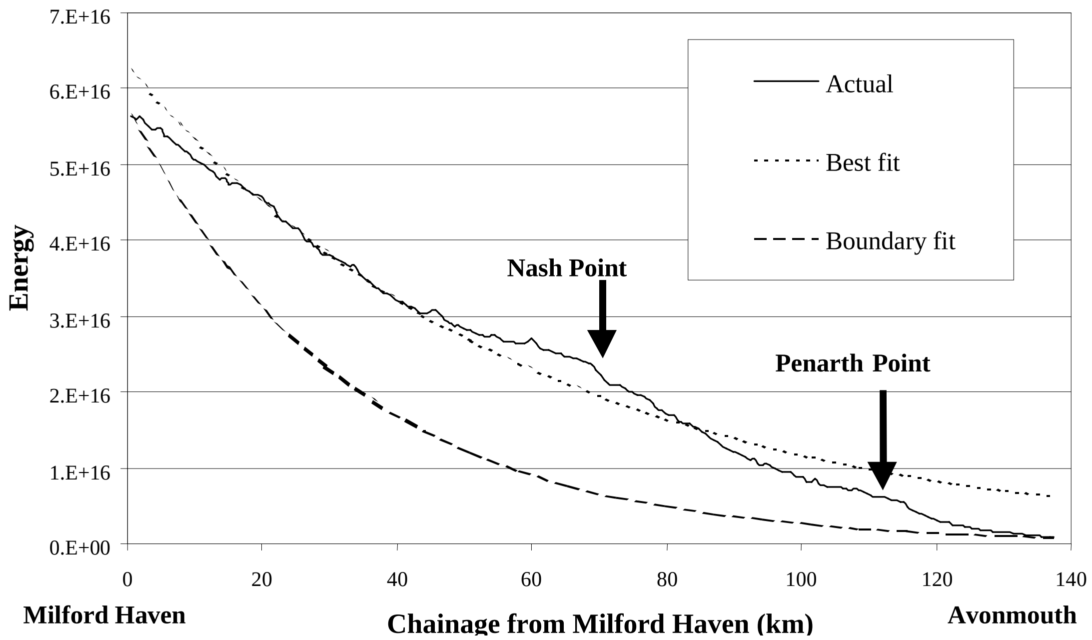

Figure 3. Variation in energy flux in the Bristol Channel

It is clear that the Boundary fit does not represent the data very well. The geology for the area and in particular the cliffs and extensive areas of exposed rock bed suggest that the section from Nash Point to Minehead may act as a constraint on the system. The section between Penarth Point and Avonmouth may also be a constraint. For further discussion on the nature of the constraints see Townend and Dun (2000).

To illustrate the influence of internal constraints, an internal boundary was introduced at the Nash Point section. The data upstream and downstream of this section were treated as separate data sets and both boundary and best fitting techniques were applied, Figure 4. Both of the fitted curves now show good agreement downstream of Nash Point. Upstream the Boundary fit continues to suggest a lower energy transmission and the Best fit overshoots beyond Penarth Point. It is likely that this can be resolved by introducing a further internal boundary at Penarth Point, which is again justified on the basis of an identifiable constraint – in this case the hard rock geology.

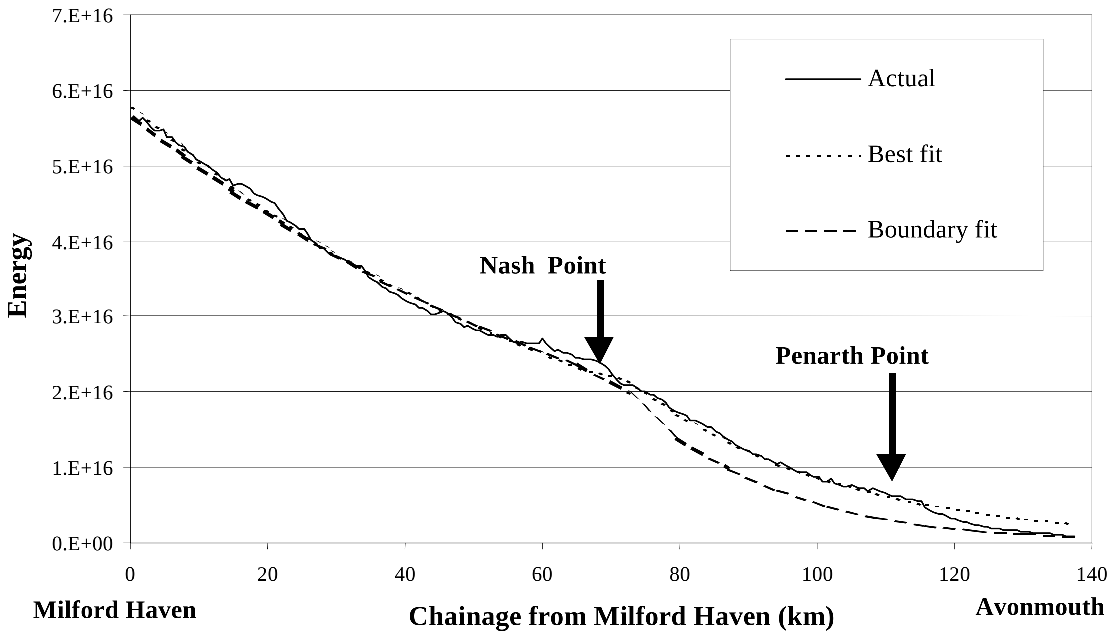

Figure 4. Variation in energy flux taking account of internal constraint

Multiple fluxes

In the derivation presented here, interest is confined to the variation in energy flux and hence the energy head and associated mass discharge. These describe the variation in the system's kinetic and potential energy. A simple extension of the method is to include the variation in water–sediment mixture density in the energy head description. There are, however, several other sources of entropy flux related to other density gradients e.g. due to sediment, salinity and temperature, which give rise to a degree of order in the system, which can be dissipated by diffusion, or reinforced and/or dissipated by convection processes. Such processes alter the order in the system and the rate of change of the irreversible processes, such as diffusion, contributes to entropy production. A more formal and complete derivation of the entropy equation will necessarily provide for these additional processes.

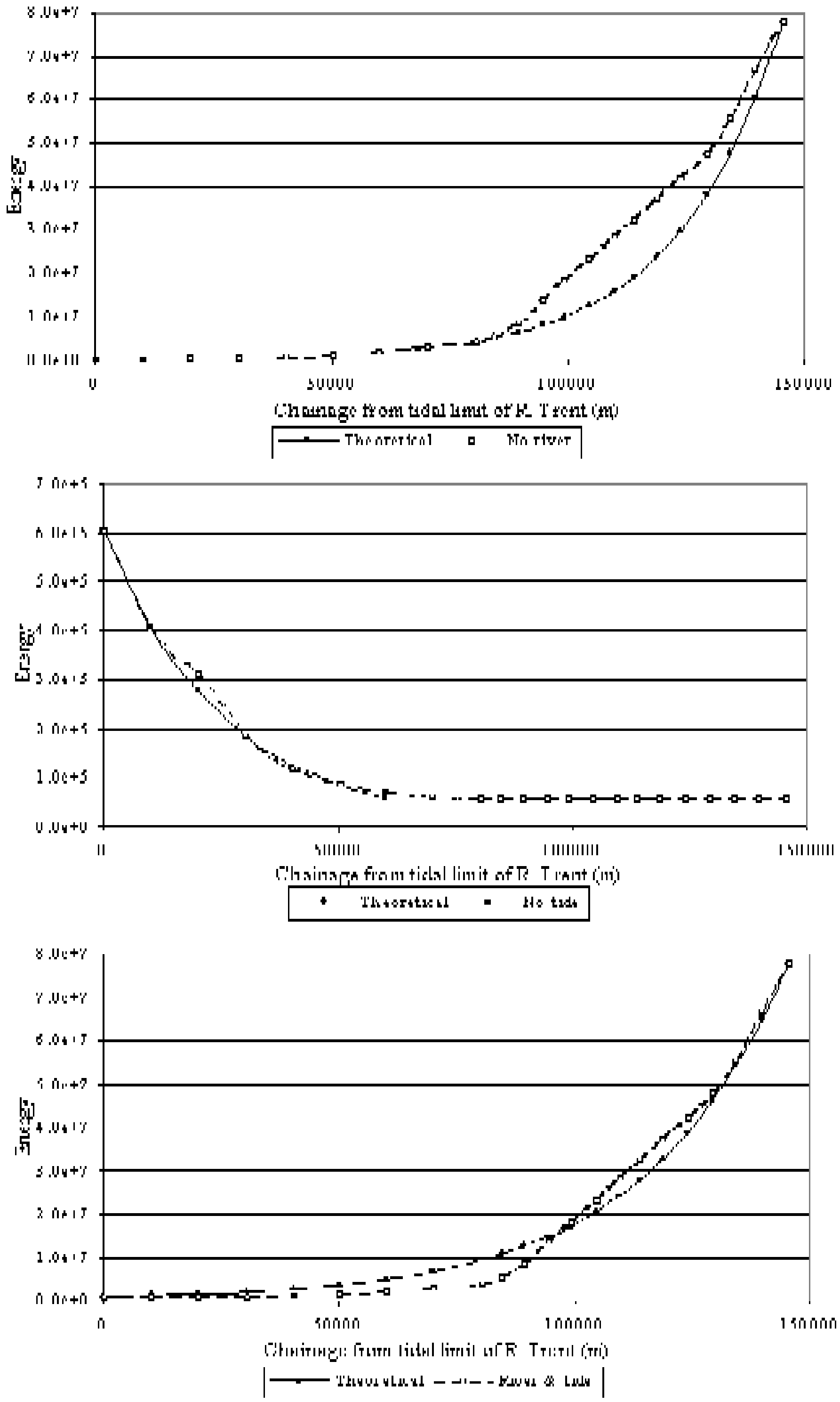

Figure 5. Variation in energy flux for tide only, river only and river and tide on the Humber

For the case of a river only, the energy flux decays from the head to an approximately constant level at a chainage of about 60km from the tidal limit. As can be seen, the curve compares well with the theoretical most probable state for the river on its own. The small deviations are thought to be a consequence of the way in which the energy from the different tributaries has been summed together. The case with a tide only decays in the opposite direction and only deviates from the river and tide case for the inner reaches of the estuary (above the 60km chainage).

The river fluxes are two orders of magnitude smaller than those due to the tide. Consequently there is only a small difference between the upper and lower plots. However the notable deviation in the outer estuary that can be clearly seen in the upper plot suggests that there are other constraints that need to be taken account of, following the approach outlined above. In this case the narrows, upstream of Hull is associated with a geological sill across the estuary and may well provide a further constraint on the system.

Considering the two cases together, there would appear a transition from fluvial to tidal dominance at about the 60km chainage. This is supported by examination of the energy head and discharge parameters, which also indicate a transition at around this location. Upstream the energy head dominates the energy term, whereas downstream the discharge makes the major contribution. This in turn is reflected in the morphology. River bed elevation governs energy for the fluvial section. In contrast, in the tidal reach there is an almost constant energy head and discharge decays exponentially. Given velocities throughout are of approximately the same magnitude (roughly 1-2m/s), this can only be achieved by the cross-sectional area varying exponentially.

This only approximately translates into an exponential decay in width. An examination of width and hydraulic depth variations along the length of the estuary reveals a strong negative correlation, particularly in the vicinity of the major bends in the system. This appears to reflect a degree of redundancy in the system which allows the width and depth to adjust to accommodate local asymmetries but maintain the longitudinal variation in cross-sectional area. More generally, the river and tidal flows can be seen as competing systems. In the outer estuary the tide dominates and in the fluvial rivers clearly the river dominates. There is however a transition zone downstream of the tidal limit where there is a progressive switch from fluvial dominance at the tidal limit over to tidal dominance. As river and tidal energy fluxes decay in opposite directions there will be an energy flux minimum in this transition zone. Changes in river flow rate, tidal range, sea level will act to move the location of this transition and system can be described as two competing components using an allometric relationship (von Bertalanffy, 1968). This approach imposes further constraints on the system and so helps reduce the uncertainty in just how the boundaries should be prescribed.

This can be illustrated by examining different forcing conditions. In Figure 6 the results for a number of different tidal and fluvial conditions are shown in terms of energy flux and percentage difference from the most probable. The tidal condition clearly has a large effect on the magnitude of the energy flux in the outer estuary but results in only small variations from the most probable (indeed the neap tide exhibits all most no difference). In contrast, the seemingly small differences in the rivers (>60km) give rise to substantial variations from the most probable. Here the surge seems to be the condition closest to the most probable state and the fluvial flood event gives rise to the largest deviation.

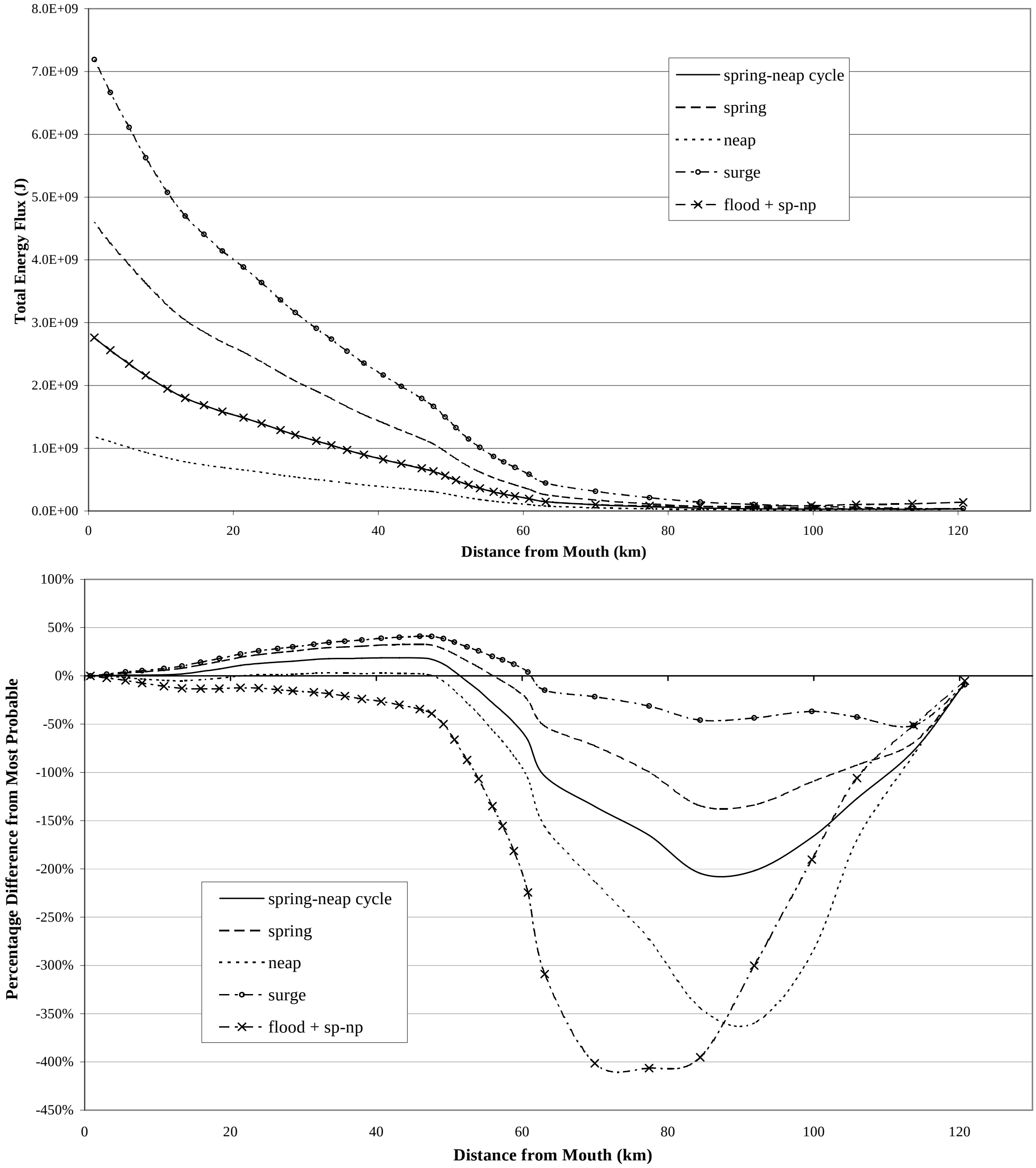

Figure 6. Energy flux as a result of different forcing conditions

The proof of minimum entropy production requires the boundary conditions to be stationary (Prigogine, 1955). The spatial and temporal scales considered therefore need to be compatible with this requirement. For an estuary as a whole one might expect the form to adjust, on average, to the mean spring-neap cycle. Variations about this, due to extreme events such as surges and fluvial floods, may then be considered as perturbations and the system response. However, in real systems we may be observing a system that has been perturbed by an extreme event and is in the process of recovering on a timescale of years or decades.

For the Humber, it has already been noted that in the rivers (>60km) the system is better adjusted to surge events than fluvial floods, Figure 6. This might be explained by the fact that surges have their greatest influence towards high water, whereas fluvial floods have a greater effect on morphology in the tidal reaches, towards low water. A perturbation due to a flood therefore moves the system away from its most probable state. However, all subsequent events (predominantly tidal) seek to restore the original state. In contrast, morphological changes due to a surge are less likely to be altered by subsequent events and the system therefore retains the capacity to accommodate these events.

This is however an area that needs further investigation. This should focus on:

- The "instantaneous state" throughout the tidal cycle and how this relates to the long-term steady state; and
- The effect of perturbations and how these relate to the long-term steady state, so providing a better means of assessing the existing condition of an estuary.

Examination of change

One particular use of the entropy technique is to consider the long-term trend of the system. Using historical data of bathymetry, sea level and tidal range, the temporal variation in the estuary relative to the most probable state can be assessed. In doing this, it is important to note that the definition of the most probable state is not constant but varies as the boundary conditions vary. Hence it is most useful to examine the variation in terms of the difference from the most probable or the percentage difference.

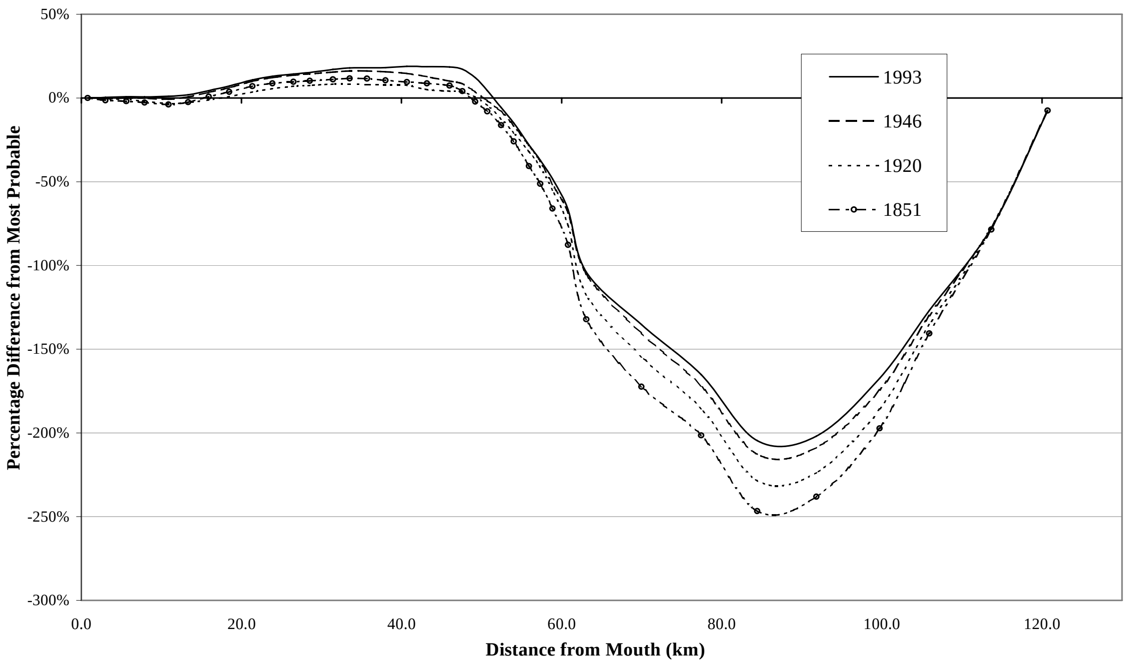

Figure 7. Historical variation for the Humber Estuary

For the Humber case study, illustrated in Figure 7, a numerical model was run using bathymetry taken in a number of different years. For each year the boundary conditions were adjusted to reflect the representative mean sea level and tidal range for the given year, based on an analysis of tidal gauge data taken at points along the estuary. Using the computed energy flux the most probable state is computed for each year, allowing the variations of absolute and relative differences to be plotted (Figure 7 shows the relative, or percentage differences). In this case it is seen that in the outer estuary there has been a small move away from the most probable state, whereas in the inner estuary (>60km) there has been a more significant move towards the most probable state. However summing the differences over the length of the system reveals a progressive reduction in the total difference. This implies that the system as a whole has progressively moved towards a more probable state.

As well as individual reaches seeking to minimise the work done, the system as a whole should also seek to do minimum work. The degree to which either or both of these requirements can be met depends on the constraints and the forcing conditions. However, using historical data this can be examined by considering the difference from the most probable for the system as a whole (Figure 8). In the case of the Humber it is seen that there is a small trend towards a more probable state. This is particularly small if the two early years are ignored (on the basis of uncertainty in the data). A more complete theoretical derivation is required in order to investigate this further.

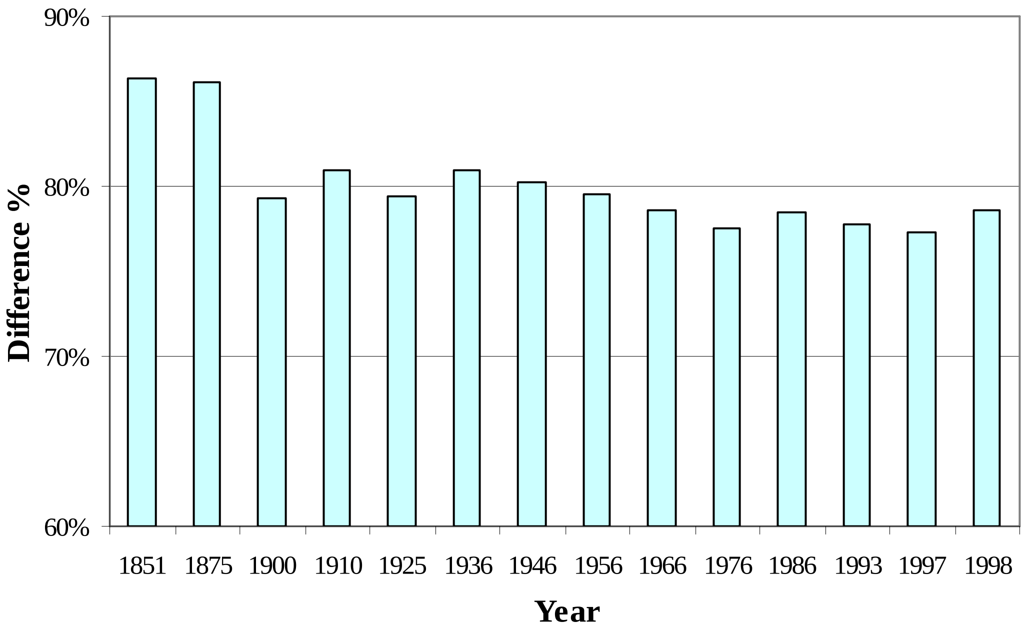

Figure 8. Difference from most probable state for the Humber Estuary as a whole

A similar approach can be used to examine the potential impact of other changes such as sea level rise or human interventions. For example, in trying to understand the likely system response to re-introducing intertidal areas within the Humber, a number of large-scale re-alignments were modelled (Townend & Pethick, 2002). This entailed the removal of the flood defences over large parts of the system (e.g. the whole of the inner estuary). The resultant percentage differences for a number of hypothetical scenarios are shown in Figure 9. At this large scale, the results suggest that the system is not particularly sensitive to the removal of defences in the outer estuary but becomes progressively more sensitive to the removal of defences further upstream.

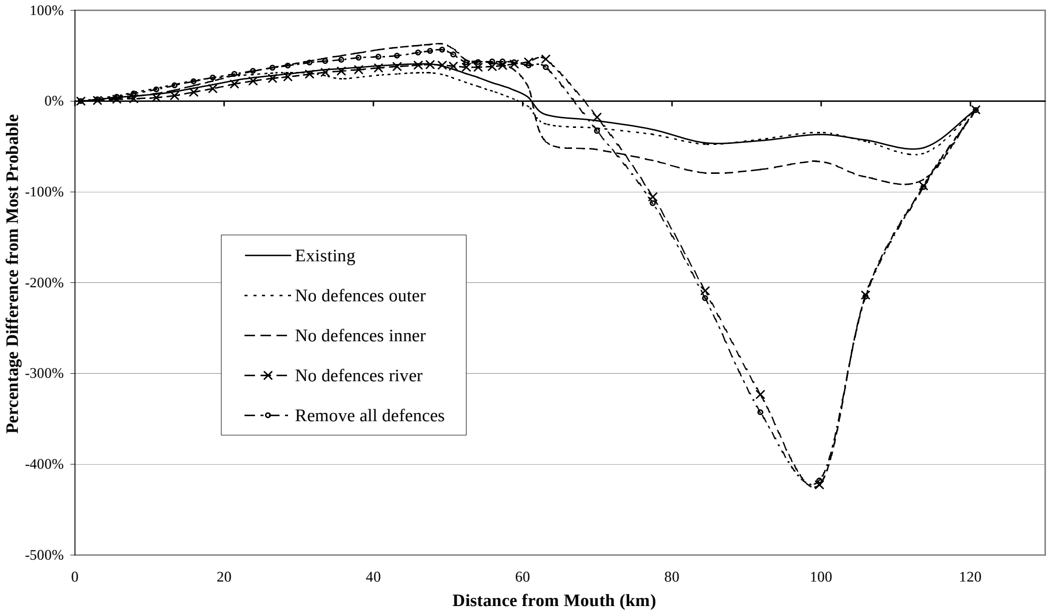

Figure 9. Influence of various large-scale interventions on the Humber Estuary

In this particular study (part of the Humber SMP studies for the Environment Agency), several smaller scale schemes were also assessed to determine the practicality and likely impact of managed realignment schemes around the estuary (Townend & Pethick, 2002). In order to provide a direct comparison of the potential sensitivity of the system to different schemes, the percentage difference from the most probable state for the system as a whole was calculated for each scenario (Figure 10). This figure shows both the sensitivity to different forcing conditions (on the left hand side) and then a range of scenarios, which were all modelled using a surge tide, revealing the greater sensitivity to schemes in the rivers (>60km) than in the outer or middle estuary (Townend & Pethick, 2002).

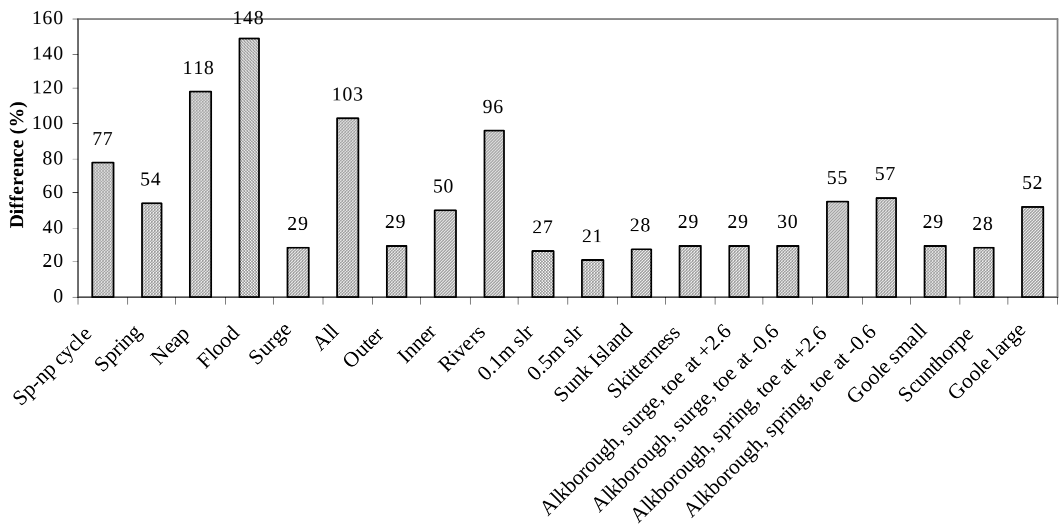

Figure 10. Difference in the total work done (for whole estuary) from theoretical most probable (%) for a range of cases (Townend & Pethick, 2002)

Summary of application

For the present, entropy based relationships cannot be used in a predictive mode. However they can be used as a diagnostic tool, along with other techniques to develop a better understanding of system behaviour and to assess the potential sensitivity to change. Further discussion of a range of applications, making use of historical and geological data can be found in Dun and Townend (1999), Gill (2000) and Townend and Pethick (2002).

When applying the method outlined in this section, it is important to recognise that:

- The boundaries need to be chosen with care to reflect the controlling inputs and constraints on the system;
- The sensitivity to different sources of energy input should be examined;
- In order to assess patterns of change consider both the internal changes and those for the system as a whole, recognising that in a steady state both should approach the most probable state; and
- Given the limitations of the current derivation and the uncertainties associated with the system dynamics, make use of relative changes rather than putting too much weight on the absolute values.

Conclusions

There is still a considerable amount of further research required to complete the theoretical derivation of the concepts that follow from the combined application of hydrodynamic and thermodynamic principles. It is hoped that this will, in time, allow a more complete determination of the goal functions and the state of a system relative to the most probable state. For now, the technique remains essentially a diagnostic tool, allowing the user to assess the condition of a system relative to some theoretical most probable state, considering just the kinetic and potential energy flux contributions. As indicated, the definition of this condition, itself requires further development. However, with sufficient historical data, or well-defined changes, such as reclamation or realignment, the technique can usefully be applied to look at the direction of change in the system as a whole.

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